5.9) $A = \begin{bmatrix} 1 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Para comprobar $Im(A) = Im(B)$ busco los autovalores de A y de B:

<u>Para A:</u>

$$P(\lambda) = \det\left(\begin{bmatrix} \lambda-1 & 0 & -i \\ 0 & \lambda-1 & 0 \\ i & 0 & \lambda-1 \end{bmatrix}\right) = -(\lambda-1).(\lambda^2-2\lambda) = -\lambda^3+2\lambda^2+\lambda^2-2\lambda$$
$$= -\lambda^3+3\lambda^2-2\lambda$$

Autovalores: $\rightarrow \lambda_1 = 0$
$\rightarrow \lambda_2 = 1$
$\rightarrow \lambda_3 = 2$

Entonces $Im(A) = (2,0,1)$

<u>Para B:</u>

$$P(\lambda) = \det\left(\begin{bmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & -1 \\ 0 & -1 & \lambda-1 \end{bmatrix}\right) = (\lambda-1).(\lambda^2-2\lambda) = \lambda^3-3\lambda^2+2\lambda$$

Autovalores: $\rightarrow \lambda_1 = 0$
$\rightarrow \lambda_2 = 1$
$\rightarrow \lambda_3 = 2$

Entonces $Im(B) = (2,0,1) = Im(A)$ ✓

PARA A

Para $\lambda=0$

$$\begin{pmatrix} -1 & 0 & -i \\ 0 & -1 & 0 \\ i & 0 & -1 \end{pmatrix} F3 \to iF1+F3 \quad \begin{pmatrix} -1 & 0 & -i \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{cases} x=-iz \\ y=0 \end{cases} \to \bar{x} = z.\underbrace{(-i,0,1)}_{\substack{\text{AUTOVECTOR} \\ \lambda=0}}$$

Para $\lambda=1$

$$\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \begin{cases} z=0 \\ x=0 \end{cases} \to \bar{x} = y.\underbrace{(0,1,0)}_{\substack{\text{AUTOVECTOR} \\ \lambda=1}}$$

Para $\lambda=2$

$$\begin{pmatrix} 1 & 0 & -i \\ 0 & 1 & 0 \\ i & 0 & 1 \end{pmatrix} F3 \to iF1-F3 \quad \begin{pmatrix} 1 & 0 & -i \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{cases} x=iz \\ y=0 \end{cases} \to \bar{x} = z.\underbrace{(i,0,1)}_{\substack{\text{AUTOVECT.} \\ \lambda=2}}$$

Normalizados los autovectores, queda la matriz U:

$$U_A = \begin{bmatrix} \frac{-i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

en donde $A = U_A . \Lambda . U_A^*$

Con $\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\boxed{\text{PARA B}}$

Para $\lambda=0$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix} F3 \to F2-F3 \quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{cases} x=0 \\ y=-z \end{cases} \to \bar{X} = z.\underbrace{(0,-1,1)}_{\substack{\text{AUTOVECTOR} \\ \lambda=0}}$$

Para $\lambda = 1$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \begin{cases} z = 0 \\ y = 0 \end{cases} \rightarrow \bar{x} = x \cdot \underbrace{(1,0,0)}_{\substack{\text{AUTOVECT.} \\ \lambda = 1}}$$

Para $\lambda = 2$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}_{F3 \to F2+F3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{cases} x = 0 \\ y = z \end{cases} \rightarrow \bar{x} = z \cdot \underbrace{(0,1,1)}_{\substack{\text{AUTOVECT.} \\ \lambda = 2.}}$$

Normalizados, queda la matriz U:

$$U_B = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$ en donde $B = U_B \cdot \Lambda \cdot U_B^*$

com $\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

S se calcula como $U_A \cdot U_B^*$

$$\rightarrow \boxed{S = \begin{bmatrix} 0 & i & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$ tal que $A = S B S^*$ ✓